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MEASUREMENT OF THE LOCAL SHEAR STRESS IN BURNER PORTS AND OTHER SHORT DUCTS BY A MASS TRANSFER TECHNIQUE

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Abstract—The local shear stress at the exit of a burner port through which fuel/air mixture flows prior to combustion has considerable influence on flame stability. For this reason a method has been developed to measure local shear stress for flow through burner ports and other short ducts in those cases where this quantity is not calculable. A model of the port is cast in either naphthalene or para-dichlorobenzene and the fuel/air flow through the burner simulated by blowing air through the model. The mass-transfer coefficient at the exit of the model port is measured using a profilometric technique and related to the shear stress in the burner using the analogy between mass and momentum transfer.

The technique has been tested by measuring the shear stress at the exit of short tubes of circular cross-section and comparing the results with theoretical solutions. Two types of tubes were investigated, those with a sharp, right angled entry and those in which the entry had been rounded. The flow through the tubes with a rounded entry remained laminar up to a length Reynolds number of about 10^5 and in this region, except for the shortest tubes investigated, the experimental results agreed fairly well with theoretical solutions for developing laminar flow. After transition to turbulent flow the experimental results agreed well with Deissler's solution for this case. The flow through the tubes with a sharp entry was turbulent over a wide range of Reynolds numbers and these experimental results were also in good agreement with the Deissler solution.

It can be concluded that the experimental technique described is of sufficient precision for the purposes for which it was developed, provided its use is restricted to conditions in which the analogy between mass and momentum transfer is valid.

NOMENCLATURE

C, concentration of diffusing species [kg/m³];
D, diameter of tube [m];
D_h, hydraulic diameter [m];
D_v, diffusivity of diffusing species [m²/s];
f_x, local friction factor based on wall shear stress;
h_D, mass-transfer coefficient [m/s];
$I_n(x)$, $i^{-n}J_n(ix)$ where $i = \sqrt{-1}$ and J_n is the nth order Bessel function;
j, mass transfer factor;
M, molecular weight of diffusing species;
m, relative boundary layer thickness δ/r_0;
N, mass flux [kg/m²s];
p_s, saturated vapour pressure of diffusing species [N/m²];
R, universal gas constant [J/kg mol deg K];
Re, Reynolds number $= Du_b\rho/\mu$;
Re_x, length Reynolds number $= xu_b\rho/\mu$;
r, radial position [m];
r_0^+, dimensionless radial position $r/(\tau_0\rho)^{\frac{1}{2}}$;
Sc, Schmidt number $= \mu/\rho D_v$;
T, absolute temperature [°K];
t, time of experiment [s];
u, axial velocity (x direction) [m/s];
u^+, dimensionless velocity $= u/(\tau_0\rho)^{\frac{1}{2}}$;
x, distance along tube [m];
y, distance normal to wall [m];

y^+, dimensionless distance normal to wall $= [(\tau_0/\rho)^{\frac{1}{2}}/\mu/\rho]y$;
z^*, dimensionless axial position $= x/DRe$.

Greek symbols
γ, dimensionless parameter;
ΔD, change in diameter [m];
δ, hydrodynamic boundary-layer thickness [m];
δ_c, mass-transfer boundary-layer thickness [m];
δ^+, dimensionless boundary-layer thickness $= [(\tau_0/\rho)^{\frac{1}{2}}\mu/\rho]\delta$;
η, relative increase in velocity of central core $(u_\delta - u_b)/u_b$;
λ, dimensionless axial velocity $= u/u_b$;
μ, viscosity [Ns/m^2];
ρ, density [kg/m^3];
σ, dimensionless axial position $= 4x/D\,Re$;
τ, shear stress [N/m^2];
v, specific volume of solid sublimate [m^3/kg].

Subscripts
B, refers to burner;
b, refers to average;
M, refers to model;
0, refers to wall ($y = 0$);
s, refers to surface;
δ, refers to outside of boundary layer.

1. INTRODUCTION

THE ADVENT of natural gas in Great Britain and the consequent conversion and redesign of appliances to burn it has re-awakened interest in the effects on flame stability of the aerodynamics of the flow of fuel/air mixture through burner ports. Recent papers [1, 2] have drawn attention to two alternative criteria for the stability of these flames with respect to blow off. Both criteria involve the velocity gradient, and hence the shear stress, at the wall at the exit of the burner port. The "boundary velocity gradient" theory of Lewis and von Elbe [3] uses the velocity gradient at the wall to characterise the velocity in the stabilising region at which the fluid velocity and local burning velocity are equal. The theory of blow-off developed by Reed [1]assumes that quenching and finally blow-off is due to excessive flame stretch caused by the shear flow in the boundary layer. Reed relates the amount of stretch to the velocity gradient at the wall through a flame stretch factor and it is suggested that when this exceeds a certain value the flame blows off. These theories and their respective merits will not be discussed further here, but if either approach is to be used for problems of burner design an accurate value for for the velocity gradient, and hence the shear stress, at the wall of burner ports is essential.

Industrial burner ports are produced in a variety of lengths, flow areas and cross-sectional shapes. The burner may be a single port or more usually a multiplicity of ports of the same or different size and shape. The entrance to the ports is generally sharp and the aerodynamics of the mixing chamber uncertain. Therefore the theoretical calculation of the shear stress at the exit of the port or ports is possible only in a limited number of instances. An experimental method is therefore required to measure either the velocity profile adjacent to the wall or the local shear stress. Various methods have recently been reviewed by Brown and Jourbert [4].

Direct measurement of velocity profiles can be made using micro-pitot tubes, hot wire anemometer probes or flow visualization techniques. These methods may be unreliable when applied to industrial burner ports, which are comparatively small and where the boundary layers are exceedingly thin.

Velocity gradients at the wall may be measured using Preston tubes [5, 6] which are small round pitot tubes resting on the wall of the duct. These tubes have been used extensively but their construction and calibration is tedious and they would be unsuited to small diameter ports.

Wall shear stress may be determined directly by measuring the force exerted on a small element. Methods have been described by

Dhawan [7], Smith and Walker [8] and Brown and Jourbert [4] amongst others, but the equipment is extremely delicate and insufficiently versatile to use for burner port measurements.

The shear stress may be determined indirectly by measuring the local heat or mass-transfer coefficient and then applying the analogies between heat, mass and momentum transfer. Local heat-transfer coefficients can be measured using thin film heated elements as described by Brown [9] and by Bellhouse and Schultz [10], and these should be suitable for the determination of wall shear stress in burner ports. However prior experience with the use of mass-transfer techniques for modelling convective heat transfer has led to the adoption of a mass transfer method in the present case. Local mass-transfer coefficients can be measured by a number of methods, and two of these have been employed by the authors and their colleagues. The two methods are an electrolytic technique, based on the measurement of the limiting diffusion controlled current to a nickel cathode, and a technique based on the sublimation of an organic solid, naphthalene or para-dichlorobenzene, into air.

Mass-transfer techniques may be used to obtain the local shear stress in two distinct ways. Firstly, small mass transfer areas enclosed by by non-transfer surfaces, may be used as the analogue of the heated film and shear stress probes. This method is particularly suited to the electrolytic technique, and has recently been described by Son and Hanratty [11]. Secondly, the walls of the whole duct may be used as the mass transfer surface and the local shear stress calculated simply from the Chilton–Colburn analogy. The sublimation technique used in this way is particularly simple and has been adopted in the present case. It is considered that this method should be widely applicable to developing flow in ducts of uniform but arbitrary crosssection. The principal advantage is that it can produce results relatively quickly with the minimum use of complex techniques and equipment.

Measurements have been made in short ducts of circular cross-section with both a sharp and a rounded entrance. Although it is true that the majority of burner ports have sharp entries, ducts with rounded entries have also been investigated because they are more amenable to theoretical analysis and there could be special circumstances in which burners with ports of this type were required. The use of such ports might be expected to result in much lower velocity gradients and thus increase the burner throughput before the onset of instability. This paper presents the theory of the mass transfer method and compares the experimental results for both sharp and rounded entry tubes with values of local shear stress predicted by various theoretical solutions.

2. RELATIONSHIP BETWEEN SHEAR STRESS AND MASS-TRANSFER COEFFICIENT

The differential equations and their boundary conditions for momentum and mass transfer across a boundary layer between a moving fluid and a solid surface are similar in cases of zero or small pressure gradients and mass transfer rates. The solution of these equations and hence the velocity and concentration profiles would therefore be expected to be similar and may be given by the general polynomials

$$\frac{u}{u_\delta} = a\left(\frac{y}{\delta}\right) + b\left(\frac{y}{\delta}\right)^2 + c\left(\frac{y}{\delta}\right)^3 + \dots \quad (1)$$

$$\frac{C - C_s}{C_\delta - C_s} = a\left(\frac{y}{\delta_c}\right) + b\left(\frac{y}{\delta_c}\right)^2 + c\left(\frac{y}{\delta_c}\right)^3 + \dots \quad (2)$$

Differentiating these equations and defining a mass-transfer coefficient as

$$N = -D_v\left(\frac{\partial C}{\partial y}\right)_0 = -h_D(C_\delta - C_s) \quad (3)$$

we obtain

$$\left(\frac{\partial u}{\partial y}\right)_0 = u_\delta \frac{h_D}{D_v}\frac{\delta_c}{\delta}. \quad (4)$$

Provided the momentum and mass-transfer boundary-layers originate from the same position the ratio of their thicknesses can be assumed to be given by

$$\frac{\delta_c}{\delta} = Sc^{-\frac{1}{3}}. \qquad (5)$$

In some simple cases this last assumption may be verified by solution of the differential equations or substitution into the boundary layer integral equations. Then

$$\left(\frac{\partial u}{\partial y}\right)_0 = u_\delta \frac{h_D}{D_v} . Sc^{-\frac{1}{3}}. \qquad (6)$$

Equation (6) can be converted to dimensionless form by multiplying by $\mu/\rho u_\delta^2$ to give

$$\left(\frac{\partial u}{\partial y}\right)_0 \frac{\mu}{\rho u_\delta^2} = \frac{h_D}{u_\delta} Sc^{\frac{2}{3}}. \qquad (7)$$

In the case of internal flows it is generally inconvenient to work in terms of the velocity and concentration at the edge of the boundary layer. In cases in which the displacement thickness is small in comparison with the characteristic dimension of the flow, these quantities may be replaced by the average velocity and concentration, defining the mass-transfer coefficient accordingly. Equation (7) then becomes

$$\left(\frac{\partial u}{\partial y}\right)_0 \frac{\mu}{\rho u_b^2} = \frac{h_D}{u_b} Sc^{\frac{2}{3}} \qquad (8)$$

or

$$\frac{\tau_0}{\rho u_b^2} = \frac{h_D}{u_b} Sc^{\frac{2}{3}}. \qquad (9)$$

This equation is usually written as

$$\frac{f_x}{2} = j. \qquad (10)$$

The final equation has also been obtained empirically and is the basis of the Chilton–Colburn analogy between mass transfer and fluid flow. The assumptions that have had to be made to derive it theoretically limit its range of applicability. The restriction to small pressure gradients confines equation (8) to such cases as flat plates and ducts of constant cross-sections in the absence of flow separation. The restriction that the displacement thickness shall be thin, eliminates such cases as fully developed laminar flow. So far as burner ports are concerned the analogy is valid in two important situations. In the first case it is valid in the hydrodynamic entry region of an internal flow where the boundary layers are still thin, and this will apply to the majority of burner ports. In the second case the analogy is applicable to fully developed turbulent flow. It is worth noting that local mass or heat transfer elements may be used over a wider range of conditions. However, for these probes the shear stress is proportional to the cube of the measured mass or heat-transfer coefficients. Experimental errors are therefore more significant than in the present method where the shear stress is directly proportional to the mass-transfer coefficient.

To relate the shear stress in a mass-transfer model to that in a burner port or duct which the model represents, the usual methods of dimensional analysis can be used to give

$$\left(\frac{\partial u}{\partial y_0}\right) \frac{\mu}{\rho u_b^2} = f\left(\frac{\rho u_b x}{\mu}, \frac{x}{D_h}\right). \qquad (11)$$

The first term in the right-hand side of this equation is the length Reynolds number, Re_x. Then

$$\left[\left(\frac{\partial u}{\partial y}\right)_0 \frac{\mu}{\rho u_b^2}\right]_M = \left[\left(\frac{\partial u}{\partial y}\right)_0 \frac{\mu}{\rho u_b^2}\right]_B \qquad (12)$$

when

$$[Re_x]_M = [Re_x]_B$$

and

$$\left[\frac{x}{D_h}\right]_M = \left[\frac{x}{D_h}\right]_B.$$

In the case of circular tubes the hydraulic diameter in this equation is of course equal to the tube diameter, D.

3. EXPERIMENTAL

The experimental technique has been developed and tested using short tubes of circular cross-section. In this case the local shear stress averaged around the circumference of the tube at the exit is determined simply by obtaining the mass-transfer coefficient from the increase in diameter of the tube during the experiment.

Model burner ports were made by casting purified molten naphthalene or para-dichlorobenzene into split cylindrical polished brass moulds of various length to inside diameter ratios. An attempt was made to ensure that the crystal structure of the subliming solid was constant for every model by always pouring the liquid at the same temperature into a cold mould. The pouring temperature was that at which crystals of solid first began to form on the surface of the melt. When cold the cast tube was removed from the mould and its average inside diameter at one end measured to the nearest micron using a micrometer microscope. The tube was then placed on the air distribution box as shown in Fig. 1, with its measured diameter uppermost. Air was passed through the tube, to simulate the flow of cold fuel/air mixture in the burner. The air temperature was noted to the nearest 0·1°C every 10 min and averaged over the duration of the experiment. After a period ranging from 1 to 3 h the tube was removed from the air box and its diameter remeasured.

Experiments were made using tubes of 1·90 cm dia. with both rounded and sharp entries. The rounded edge ports were made by using mould cores which had been machined to the desired shape, the curvature being formed from a quadrant of a circle whose radius was half the radius of the tube. The length of each tube was taken as the distance between the entrance and exit planes. The sublimation of the solid inevitably causes the tube dimensions and inlet conditions to change continually. In particular a sharp entry will become rounded. Sharp entry tubes were therefore ground flat before each experiment and a newly cast tube used as soon as the inlet conditions of any tube became significantly changed.

The time during which air was passed through the model was chosen to ensure an increase in diameter of about 300 μ. For low flow rates and, consequently, low transfer rates, para-dichlorobenzene was used as the subliming solid and for

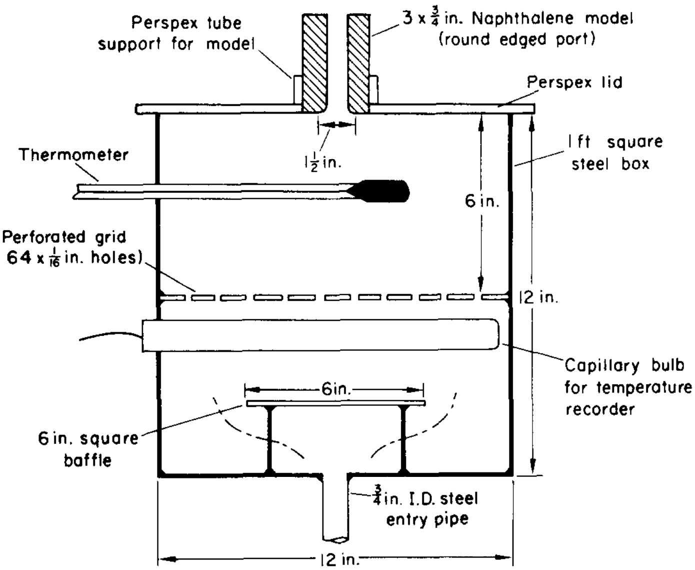

FIG. 1. Diagram of apparatus.

higher flow rates naphthalene was used. The choice of solids was such that the duration of an experiment was restricted to a period of 1–3 h. The measurements of the diameters were made as quickly as possible to avoid errors due to the natural sublimation of the solids into the atmosphere. Experiments on the rate of natural convection showed that the losses were negligible compared with an increase in diameter of 300 μ when the total time taken to measure the diameter before and after the experiment was ½ h.

The total weight loss from the inside of the tube was recorded during each experiment. The corresponding partial pressure of sublimate in the air stream was calculated and found to be negligible compared with the saturated vapour pressure at the air temperature.

4. CALCULATION OF MASS-TRANSFER COEFFICIENT AND SHEAR STRESS

The mass-transfer coefficient is calculated from the increase in diameter of the naphthalene or para-dichlorobenzene tube from the equation

$$h_D = \frac{(\Delta D)}{v2tC_s - C_b)}. \qquad (13)$$

Assuming that the average concentration, C_b, is negligible compared with the surface concentration, C_s, and expressing the surface concentration in terms of partial pressure, equation (13) becomes

$$h_D = \frac{RT(\Delta D)}{v2tMp_s}. \qquad (14)$$

The vapour pressure of para-dichlorobenzene has been measured by Darkis *et al.* [12] who give the equation

$$\log_{10} p_s = 11{\cdot}985 - \frac{3570}{T}. \qquad (15)$$

These measurements have been confirmed by Bedingfield and Drew [13], Roark and Nelson [14], Walsh and Smith [15] and Adams and McFadden [16]. The International Critical Tables give vapour pressure data which are 30–40 per cent lower than those represented by equation (15).

The vapour pressure of naphthalene has been measured by Sherwood and Bryant [17] who give the equation

$$\log_{10} p_s = 11{\cdot}55 - \frac{3765}{T}. \qquad (16)$$

Christian and Kezios [18] analysed data from several sources and presented an expression which is in good agreement with equation (16).

In order to calculate the shear stress from the mass transfer coefficient the Schmidt number and hence the diffusivity must be known. There appears to be no direct measurement of the diffusivity of either para-dichlorobenzene or naphthalene in air quoted in the literature. Ricetti and Thodos [19] calculated the diffusivity of para-dichlorobenzene according to an equation of Hirschfelder, Curtiss and Bird [20] and obtained a diffusion coefficient of $7{\cdot}15 \times 10^{-6}$ m²/s and a Schmidt number of 2·24 at 28°C. Bedingfield and Drew [13] used the Gilliland equation to calculate a diffusivity of $5{\cdot}89 \times 10^{-6}$ m²/s and a Schmidt number of 2·23 at 0°C. For the calculations described in the paper the Schmidt number of the para-dichlorobenzene/air system has been taken to be constant at 2·24.

Christian and Kezios [18] used the Gilliland equation to calculate the diffusivity of the naphthalene and air system. Sherwood and Trass [21] used equations given by Hirschfelder *et al.* [20] to calculate the diffusivity and viscosity, and the ideal gas law to calculate the density. They then obtained the following expression for the Schmidt number of the naphthalene/air system

$$Sc = 7{\cdot}803\, T^{-0{\cdot}185}. \qquad (17)$$

This equation has been used to calculate the results discussed in this paper·

5. THEORETICAL SOLUTIONS FOR LAMINAR FLOW IN THE ENTRANCE REGION OF TUBES WITH CIRCULAR CROSS-SECTION

Numerous applications of the equations of motion to laminar flow in the entrance regions of tubes have been reported in the literature, and the solutions described below are usually accepted as being amongst the most reliable. These solutions were originally expressed in terms of velocity profiles and apparent friction factors, and from them the corresponding values of local shear stress have been derived. The equations below express the product of the local friction factor and the Reynolds number as a function of a dimensionless length factor. The local shear stress is then obtained from:

$$f_x Re = \frac{2D\tau_0}{u_b \mu}. \qquad (18)$$

The corresponding velocity gradient at the wall is obtained from

$$f_x Re = \frac{2D}{u_b}\left(\frac{\partial u}{\partial y}\right)_0 \qquad (19)$$

The dimensionless group $f_x Re$ has a value of 16 for fully developed laminar flow.

5.1 *Langhaar solution*

Langhaar [22] linearised the momentum equation for laminar flow to the form

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} - \beta^2 u = \alpha \qquad (20)$$

where α and β are functions of x alone.

The solution of this equation gives the velocity profile in the form

$$\lambda = [I_0(\gamma) - I_0(\gamma q)]/I_0(\gamma) \qquad (21)$$

where λ is a dimensionless axial velocity, q a dimensionless radial position and γ a dimensionless parameter which is a function of a dimensionless axial co-ordinate σ. The parameter σ is defined by

$$\sigma = \frac{4L}{D\,Re}. \qquad (22)$$

The local friction factor, and hence the shear stress, is given by

$$f_x Re = \frac{2\gamma^2}{I_2^2(\gamma)}[I_1^2(\gamma) - I_0(\gamma)I_2(\gamma)]. \qquad (23)$$

Langhaar evaluated γ for a range of values of σ and these have been used to calculate the corresponding friction factors from equation (23).

5.2 *Schiller solution*

Schiller [23] developed an integral method in which the velocity profiles were assumed to be straight line segments terminated by parabolic arcs. The solution was obtained by applying the momentum equation to the whole body of the fluid and the Bernoulli equation to the central core. The local friction factor is given by

$$f_x Re = \frac{8(1+\eta)}{2 - \left(4 - 6\dfrac{\eta}{1+\eta}\right)^{\frac{1}{2}}} \qquad (24)$$

where η is a dimensionless central core velocity given by

$$\eta = \frac{u_\delta - u_b}{u_b}. \qquad (25)$$

Schiller evaluated η as a function of the dimensionless length parameter σ and these values have been substituted into equation (24) to obtain the corresponding friction factors.

5.3 *Campbell and Slattery solution*

Campbell and Slattery [24] improved the Schiller solution by taking into account viscous dissipation which Schiller had ignored. The macroscopic mechanical energy balance (rather than the Bernoulli equation), was applied to the central core. The local friction factor is then given by

$$f_x Re = \frac{48}{m(m^2 - 4m + 6)} \qquad (26)$$

where m is the dimensionless boundary-layer thickness defined by

$$m = \delta/r_0. \qquad (27)$$

Campbell and Slattery evaluated m for various values of x/DRe and these values have been used to calculate $f_x Re$.

5.4 *Shapiro, Siegel and Kline solutions*

Shapiro *et al.* [25] followed Schiller in neglecting viscous dissipation but used more refined forms for the assumed velocity profile. The boundary layer velocity profile was assumed to transform, as the boundary layer thickens, from a velocity profile characteristic of flow over a flat plate to a parabolic profile which is characteristic of fully developed flow. Two forms of the flat plate velocity profile were assumed. These were the cubic parabola and the Polhausen velocity profile.

From the cubic parabola assumption the local friction is given by

$$f_x Re = \frac{120(m+3)}{m(60 - 45m + 17m^2 - 2m^3)}. \quad (28)$$

From the Pohlhausen velocity profile assumption the local friction factor is given by

$$f_x Re = \frac{240}{m(30 - 18m + 2m^2 + m^3)} \quad (29)$$

where m is again the dimensionless boundary-layer thickness defined by equation (27).

Shapiro *et al.* evaluated m for a range of values of x/DRe for both velocity profile assumptions and these values have been used to calculate the corresponding local friction factors.

5.5 *Comparison of solutions*

The local friction factors predicted by the different theories have been calculated by the authors and the results tabulated elsewhere [26]. A comparison of the predictions is shown graphically in Fig. 2 where the product of the local friction factor and the Reynolds number is plotted against the parameter $Re\,D/x$. The agreement between the integral solutions is close but they differ significantly from the differential solution of Langhaar. It is worth noting that when the various solutions are compared in terms of pressure drop and velocity profile the agreement between them is much closer than is the case when wall shear stress is the basis for comparison.

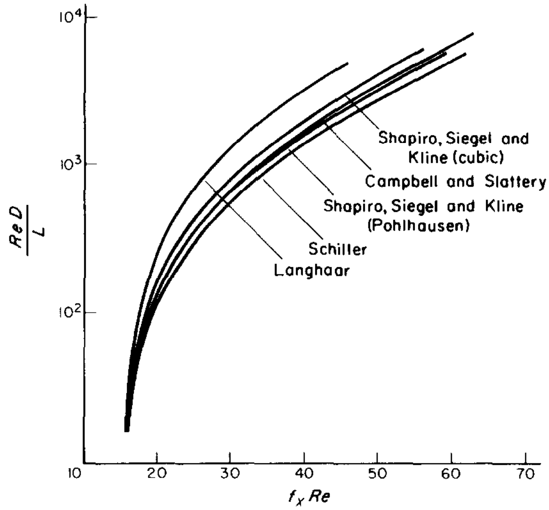

FIG. 2. Solutions for local friction factors—laminar flow.

Many authors [22, 24, 25, 27] have compared the theoretical solutions with each other and with experimental results. However, these comparisons have been confined to pressure drop and velocity profile and in these respects no conclusive evidence has been presented for the superiority of any one solution over the others.

6. SOLUTION FOR TURBULENT FLOW IN THE INLET REGION OF TUBES WITH CIRCULAR CROSS SECTION

The developing turbulent boundary-layer in the inlet region of a circular tube has been considered by Deissler [28], who gives the following boundary layer integral equations

$$\frac{x}{D} = \int_{Re/2}^{u_\delta^+ r_0^+} \left[\frac{1}{4}\frac{\delta^+}{r_0^+}\frac{1}{r_0^+}(2 - \delta^+/r_0^+)\,u_\delta^+ - \frac{1}{2r_0^{+3}}\int_0^{\delta^+} u^+(r_0^+ - y^+)\,\mathrm{d}y^+ \right] \mathrm{d}(r_0^+ u_\delta^+) + \int_0^{[]} \frac{1}{2r_0^{+2}}\,\mathrm{d}\left[\int_0^{\delta^+} (u_\delta^+ - u^+)\,u^+(r_0^+ - y^+)\,\mathrm{d}y^+\right]. \quad (30)$$

$$Re = \frac{4}{r_0^+} \int_0^{\delta^+} u^+(r_0^+ - y^+)\,dy^+ + \frac{2u_\delta^+(r_0^+ - \delta^+)^2}{r_0^+}. \quad (31)$$

These equations may be solved, if the velocity profile is specified, to yield values of x/D for chosen values of Re and δ^+. The corresponding local friction factor is then calculated from the following equation

$$f_x Re = \frac{r_0^{+2}}{Re}. \quad (32)$$

The velocity profile suggested by Deissler is obtained from two separate equations for the laminar and turbulent layers. Spalding [29] has suggested the following continuous equation to fit the entire boundary layer

$$y^+ = u^+ + \frac{1}{E}\left[e^{ku^+} - 1 - ku^+ - \frac{(ku^+)^2}{2!} - \frac{(ku^+)^3}{3!} - \frac{(ku^+)^4}{4!}\right] \quad (33)$$

where $k = 0{\cdot}407$ and $E = 10$.

Equations (30)–(33) have been solved numerically for a range of Reynolds numbers and x/D ratios. The results have been tabulated [26] and are shown as solid lines in Fig. 3, the parameter of the curves being the Reynolds number. Also shown in the figure, as broken lines, are the friction factors given by the Blasius equation for fully developed turbulent flow.

$$f_x Re = 0{\cdot}0792\, Re^{0{\cdot}75}. \quad (34)$$

The Deissler solutions are asymptotic to this equation at large x/D ratios above Reynolds numbers of about 6×10^3.

The Deissler solution was derived for the case where the boundary layer is turbulent throughout the tube. Deissler points out that for flow in which the boundary layer is partially laminar, or for cases in which there are large disturbances at the entrance, such as might be caused by a right angled-edge entrance, the analysis may not be correct.

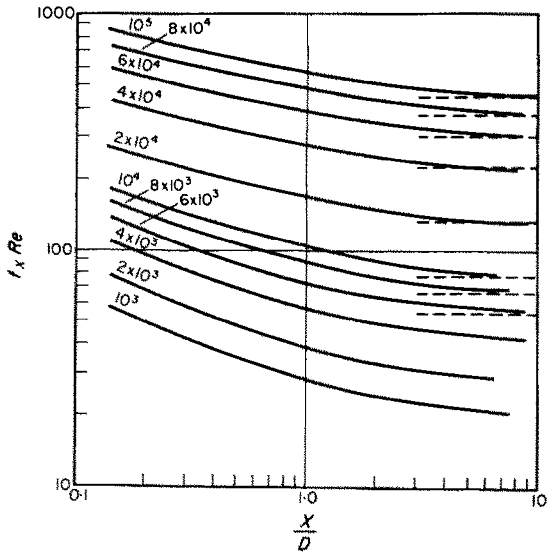

FIG. 3. Solution for local friction factors—turbulent flow.

7. RESULTS

Experiments were carried out using tubes of 1·90 cm dia. with length to diameter ratios of $\frac{1}{2}$, 2 and 4. Length to diameter ratios of about one half represent the shortest ports that are generally encountered in industrial practice although the majority of burner ports formed by drilling circular holes in standard sizes of metal tubing will have length to diameter ratios in the range 1–4. The experimental results are shown in Figs. 4–6 where the j factor for mass transfer ($f_x/2$) is plotted against the length Reynolds number. Also included in these figures are lines indicating the theoretical solutions for developing laminar and turbulent flow in tubes. The Campbell and Slattery analysis is used to typify the various laminar flow solutions, and the Deissler solution is shown for turbulent flow. The agreement between the results obtained using naphthalene and para-dichlorobenzene is good in the region in which they overlap. For the sake of clarity no distinction is made in Figs. 4–6 between the results obtained using the two solids.

8. DISCUSSION

The most striking feature of Figs. 4–6 is the difference between the results for tubes with sharp and rounded entries and these two cases will be discussed separately.

The results for the rounded entry tubes show a marked transition to turbulent flow at length Reynolds numbers ranging from 3×10^4 to 10^5. These values are generally lower than those noted by Shapiro and Smith [30] who found transition at about $Re_x = 10^5$ for both air and water. However, transition is affected by initial turbulence level and entrance shape as well as by the Reynolds number. Hot wire anemometer measurements showed that the free stream turbulence intensities were of the order of 1–2 per cent.

The results for laminar flow through the rounded entry tubes with x/D ratios of 2 and 4 are close to the relevant theoretical solutions, but the scatter of the results is as great as the differences between the solutions and no conclusion can be drawn as to which solution is to be preferred. The results for laminar flow through tubes with x/D ratio of $\frac{1}{2}$ are parallel to the theoretical solutions but considerably lower. For tubes as short as these the curved entry becomes important and it may be argued that the length term should not be taken as the distance between the inlet and exit planes. If based on the length of the constant cross section portion of the tubes then the agreement between the theory and the experiments becomes even worse and it is only marginally better if the length of the flow path is taken as the characteristic dimension. The experimental results for the tubes with x/D ratio of $\frac{1}{2}$ consequently cast doubt on either the mass transfer analogy or the theoretical solutions for very short tubes. Unfortunately both the solutions and the analogy are less liable to be valid in this region. The theoretical solutions ignore radial velocity components produced both by the curved entry and also by normal boundary layer growth. Various finite difference procedures for solving the Navier–Stokes equations taking into account radial velocity components have been proposed [31, 32] and these may be more applicable than the boundary layer integral methods close to the tube entrance. The mass transfer analogy is liable to break down near the entrance due to the existence of pressure gradients which may not be negligible in this region. The experimental results show that there is very little in-

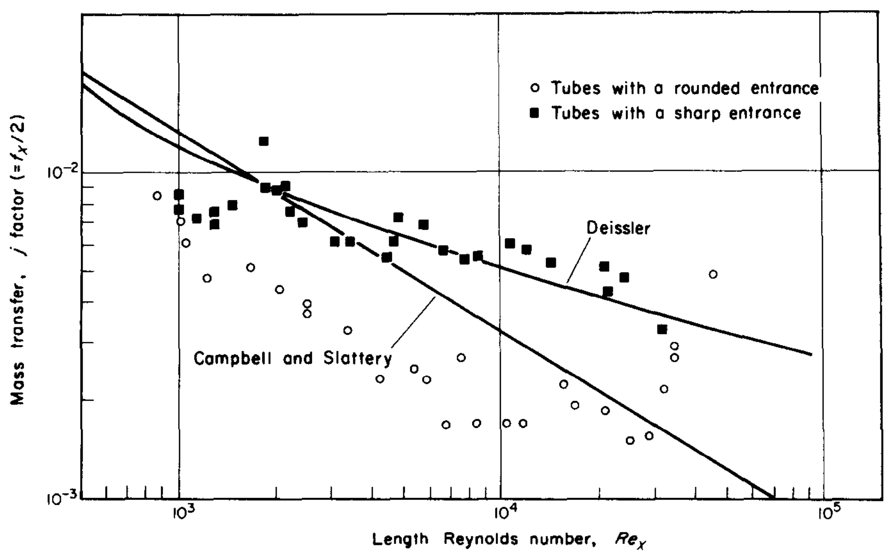

FIG. 4. Experimental results, length to diameter ratio $\frac{1}{2}$.

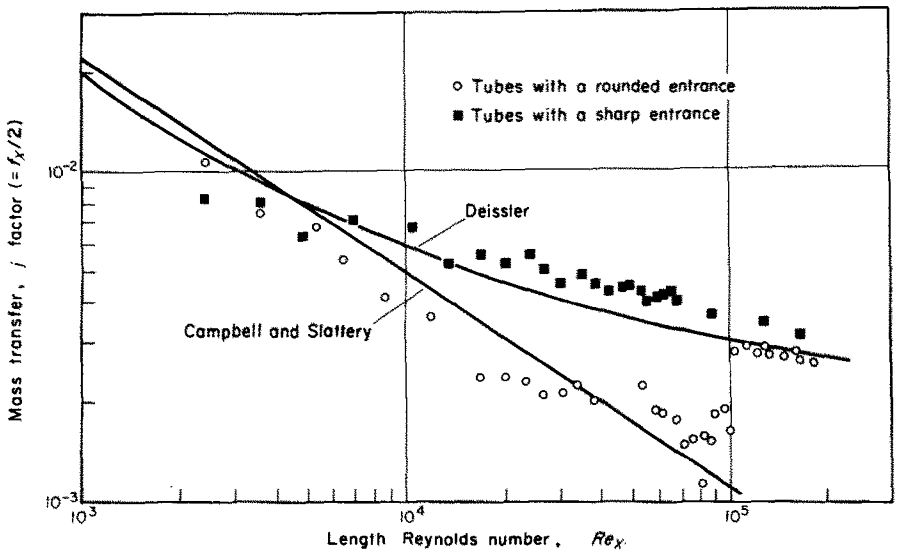

FIG. 5. Experimental results, length to diameter ratio 2.

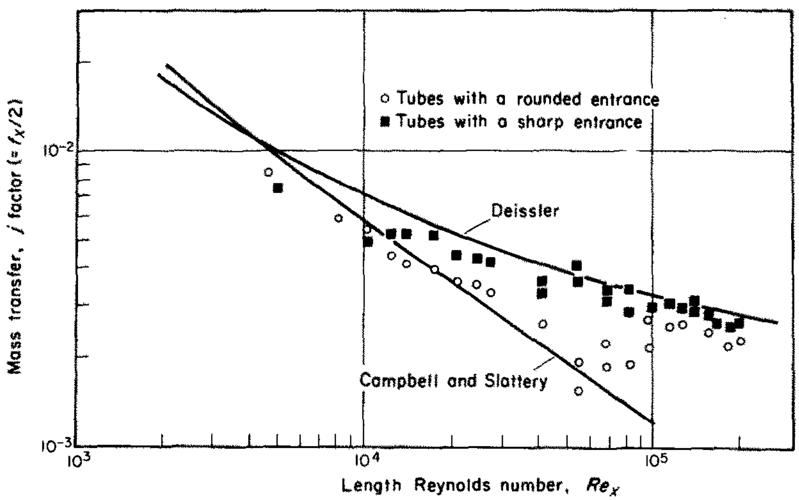

FIG. 6. Experimental results, length to diameter ratio 4.

crease in mass transfer rates over the x/D range of $4\frac{1}{2}$.

When the flow through the rounded entry tubes becomes turbulent the results agree well with the Deissler solution for developing turbulent flow. It would be expected that the analogy is most valid under these conditions and the agreement between theory and experiment is encouraging.

The results for the sharp entry tubes indicate turbulent flow over a wide range of Reynolds numbers. The turbulence is undoubtedly produced by flow separation at the sharp corner followed by reattachment some distance down-stream. Reattachment probably occurs about one diameter from the entrance [33]. For the tubes of length 2 and 4 diameters therefore the exit conditions are those of a developing turbulent boundary layer under which conditions the mass transfer analogy would be expected to be valid. The experimental results for for these tubes are close to but slightly higher than those predicted by Deissler for turbulent flow. They are also a few per cent higher than the equivalent turbulent flow results for the rounded entry tubes. This would be expected since the turbulent boundary layer grows from the point of reattachment and not the tube entrance for sharp entry tubes. It would appear that although Deissler's solution neglects separation it still predicts wall-shear stress fairly well at 2 and 4 diameters from the tube entrance despite separation and reattachment having occurred.

In the case of sharp entry tubes with a length to diameter ratio of $\frac{1}{2}$ there is some evidence of either transition from laminar to turbulent flow or the onset of separation at a length Reynolds number of about 1500. At higher Reynolds numbers it is probable that reattachment does not occur before the tube exit is reached. The mass-transfer results were therefore obtained in uncertain flow conditions which may vary with Reynolds number. In some cases the flow will be reversed and hence these results must be treated with caution. It is apparent that a closer understanding of conditions at the wall for flow through sharp entry tubes would require measurements of wall static pressure and flow visualisation.

Thus far, the method has been applied only to ducts of circular cross-section, in order to test its validity. The method was however, developed for the purpose of measuring the shear stress under entry conditions in cases where the theoretical solutions are less certain; these include tubes of more complex cross-section, e.g. triangular ducts, and situations in which the entrance conditions are non-uniform. In these circumstances many more measurements of the change in tube dimensions are required to determine local mass-transfer coefficients. If the measurements are made directly on the mass transfer models then the error resulting from natural sublimation can be excessive. For this reason it is suggested that the end of the tube be photographed and the change in dimensions obtained from measurements on the negative

9. CONCLUSIONS

The method described in this paper gives a clear indication of transition from laminar to turbulent flow conditions. In rounded entry tubes the flow remained laminar in the cases investigated up to length Reynolds numbers of about 10^5. In this régime the wall shear stresses calculated from the mass transfer results agree fairly well with the theoretical solutions for x/D ratios of 2 and 4 but lie lower than the theoretical solutions for tubes with x/D ratio of $\frac{1}{2}$. Unfortunately the precision of the experiments in the laminar regime is not very high and the scatter of the results is as great as the differences between the various predictions. After transition to turbulent flow the experimental results for the rounded entry tubes are close to Deissler's solution for this case.

In the case of tubes with a sharp entry the flow is turbulent over a wide range of Reynolds numbers. Under these conditions the results again agree well with the Deissler solution despite the probable occurrence of separated flow. The results for $x/D = \frac{1}{2}$, however, must be treated with some reserve since it is not certain that the mass-transfer analogy is valid under these conditions.

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MESURE DE LA CONTRAINTE LOCALE DE CISAILLEMENT DANS DES ORIFICES DE BRÛLEURS ET D'AUTRES CONDUITS COURTS PAR UNE TECHNIQUE DE TRANSPORT DE MASSE

Résumé—La contrainte locale de cisaillement à la cortie d'un orifice de brûleur à travers lequel un mélange combustible/air s'écoule avant la combustion a une influence considérable sur la stabilité de flamme. Pour cette raison, une méthode a été élaborée pour mesurer la contrainte locale de cisaillement pour l'écoulement à travers des brûleurs et d'autres conduits courts dans les cas où cette quantité n'est pas calculable. Un modèle de l'orifice est moulé dans du naphthalène ou du paradichlorobenzène et l'écoulement combustible/air à travers le brûleur est simulé en soufflant de l'air à travers le modèle. Le coefficient de transport de masse à la sortie de l'orifice du modèle est mesuré à l'aide d'une technique profilométrique et reliée à la contrainte de cisaillement dans le brûleur en employant l'analogie entre le transport de masse et celui de quantité de mouvement.

La technique a été essayée en mesurant la contrainte de cisaillement à la sortie de tubes courts de section circulaire et en comparant les résultats avec les solutions théoriques. Deux types de tubes ont été étudiés, ceux avec une entrée aux arêtes vives et à angle droit et ceux pour lesquels l'entrée a été arrondie. L'écoulement à travers les tubes avec une entrée arrondie restait laminaire jusqu'à un nombre de Reynolds basé sur la longueur d'environ 10^3 et, dans cette région, excepté pour les tubes étudiés les plus courts, les résultats expérimentaux étaient en bon accord avec les solutions théoriques pour un écoulement laminaire en train de s'établir. Après la transition vers l'écoulement turbulent, les résultats expérimentaux étaient en bon accord avec la solution de Deissler pour ce cas. L'écoulement à travers les tubes avec une entrée aux arêtes vives était turbulent dans une large gamme de nombres de Reynolds et ces résultats expérimentaux étaient aussi en bon accord avec la solution de Deissler.

On peut conclure que la technique expérimentale décrite est d'une précision suffisante pour les buts pour lesquels elle avait été établie, pourvu que son emploi soit restreint aux conditions dans lesquelles l'analogie entre le transport de masse et celui de quantité de mouvement est valable.

MESSUNGEN DER ÖRTLICHEN SCHUBSPANNUNG IN BRENNERÖFFNUNGEN UND ANDEREN KURZEN KANÄLEN MIT HILFE DES STOFFÜBERGANGS

Zusammenfassung—Die lokale Schubspannung am Austritt eines Brennerkanals, durch den ein Brennstoff-Luft-Gemisch zur Verbrennung strömt, hat einen beträchtlichen Einfluss auf die Flammenstabilität. Darum wurde eine Methode entwickelt um lokale Schubspannungen bei Strömungen in Brenneröffnungen und anderen kurzen Kanälen in solchen Fällen zu messen, wo die Berechnung versagt. Aus Naphtalin oder Paradichlorbenzol wurde ein Modell des Kanals gegossen, und die Brennstoff-Luft-Strömung durch den Kanal wurde im Modell ersetzt durch einen Kuftstrom. Der Koeffizient des Stoffübergangs am Austritt des Modellkanals wurde mit Hilfe einer profilometrischen Technik gemessen und unter Benutzung der Analogie zwischen Stoff- und Impuls-Austausch in Beziehung gesetzt zur Schubspannung im Kanal.

Die Methode wurde durch Messung der Schubspannung am Austritt von kurzen kreisrunden Rohren und durch Vergleich dieser Ergebnisse mit theoretischen Lösungen getestet. Es wurden zwei Rohrtypen untersucht, solche mit einem scharfen, rechtwinkeligen Eintritt, und solche, bei denen die Mündung gerundet war. Die Strömung durch die Rohre mit abgerundetem Eintritt blieb laminar bis zu Reynoldszahlen (gebildet mit der Rohrlänge) von etwa 10^5 und in diesem Bereich stimmten mit Ausnahme der kürzesten untersuchten Rohre, die experimentellen Ergebnisse ziemlich gut mit den theoretischen Lösungen der laminaren Anlaufströmung überein. Nach dem Übergang in turbulente Strömung stimmten die experimentellen Ergebnisse gut mit der Lösung von Deissler für diesen Fall überein. Die Strömung durch die Rohre mit scharfkantigem Eintritt war über einen weiten Bereich der Reynoldszahlen turbulent und auch diese experimentellen Ergebnisse waren in guter Übereinstimmung mit der Lösung Deisslers.

Man kann daraus folgern, dass die beschriebene Experimentiertechnik für den beabsichtigten Zweck von ausreichender Genauigkeit ist, wobei zu berücksichtigen ist, dass die Anwendung auf die Fälle beschränkt ist, bei denen die Analogie zwischen Stoff- und Impulsaustausch gültig ist.

ИЗМЕРЕНИЕ ЛОКАЛЬНОГО НАПРЯЖЕНИЯ СДВИГА В ОТВЕРСТИЯХ ГОРЕЛОК И В ДРУГИХ КОРОТКИХ ТРУБОПРОВОДАХ С ПОМОЩЬЮ ПЕРЕНОСА МАССЫ

Аннотация—Локальное напряжение сдвига на выходе из отверстия горелки, через которое проходит смесь топлива с воздухом до наступления горения, оказывает значительное влияние на стабильность пламени. В связи с этим разработан метод

измерения локального напряжения сдвига для течения через горелку и другие короткие трубопроводы в тех случаях, когда эта величина не поддается расчетам. Модель отверстия отливалась в нафталине или парадихлорбензине, и поток топлива/воздуха через горелки имитировался продуванием воздуха через модель. Коэффициент диффузии на выходе из модели отверстия измеряли профилометрическим методом и относили его к напряжению сдвига в горелке, используя аналогию между переносом массы и количества движения. С целью проверки результатов проводилось измерение напряжения сдвига на выходе из коротких трубок круглого поперечного сечения, и данные сравнивались с результатами теоретических решений. Исследовалось два типа трубок: с острым под прямым углом входом и с закругленным входом. Течение в трубке с закругленным входом оставалось ламинарным до значения критерия Рейнольдса, рассчитанного по длине трубки, до 10^3 и в этой области, за исключением самых коротких коротких исследованных трубок, экспериментальные результаты хорошо согласовывались с теоретическими решениями для развитого ламинарного течения. После перехода к турбулентному течению экспериментальные результаты хорошо согласовывались с решением Дайсслера для данного случая. Течение через трубки с острым входом было турбулентным в широком диапазоне чисел Рейнольдса, и эти экспериментальные результаты также хорошо согласовывались с решением Дайсслера.

Можно сделать вывод, что описанная экспериментальная техника является достаточно точной для тех целей, для которых она была разработана, при условии, что её применение ограничено условиями, в которых справедлива аналогия между переносом массы и количества движения.